Chapter 1 to 8

/60 marks

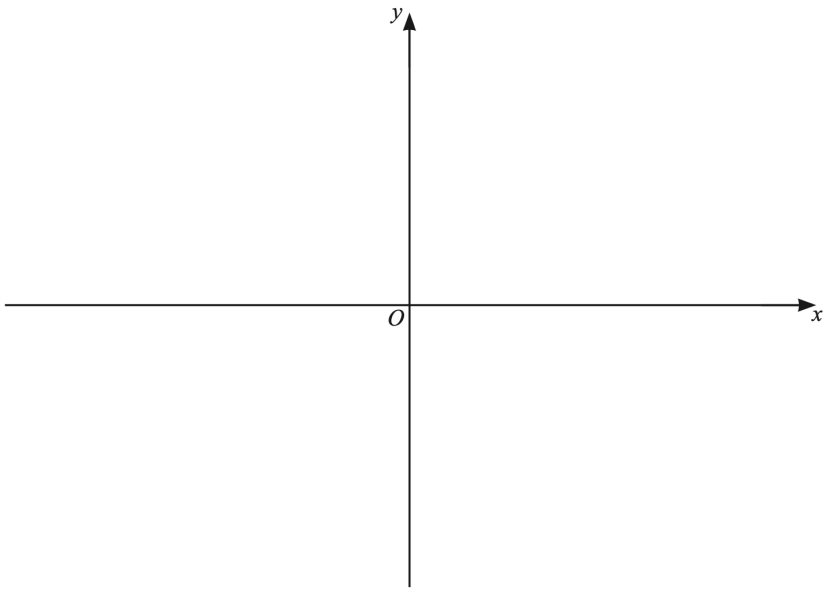
1. (a) Show that $2x^2 + x - 15$ can be written in the form $2(x + a)^2 + b$, where *a* and *b* are **exact** constants to be found.

[2]

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$.

[2]

(c) On the axes, sketch the graph of $y = |2x^2 + x - 15|$, stating the coordinates of the points where the graph meets the coordinate axes.



[3]

(d) Write down the value of the constant *k* for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions.

[2]

2. (a)Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$

$$xy = \frac{1}{2}$$

[3]

(b) Solve the equation, $log_3 x + 3 = 10\, log_x 3$, giving your answers as powers of 3.

[4]

3. A function f(x) is such that $f(x) = e^{3x} - 4$, for $x \in R$.

a. Find the range of f.

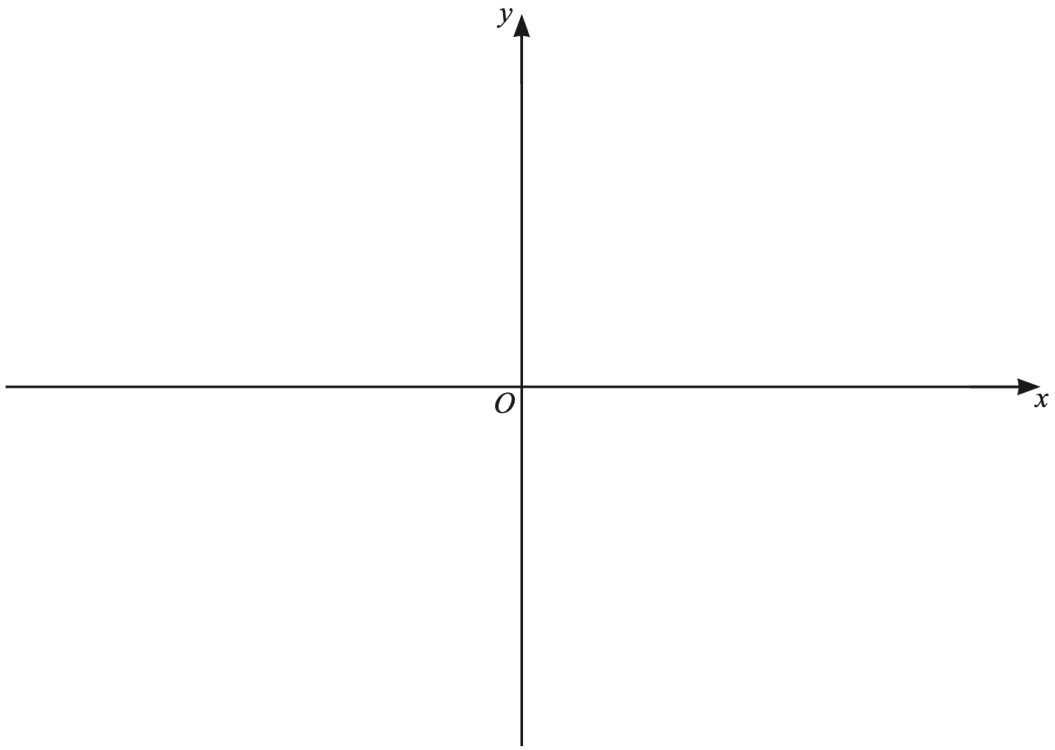
[1]

b. Find an expression for $f^{-1}(x)$.

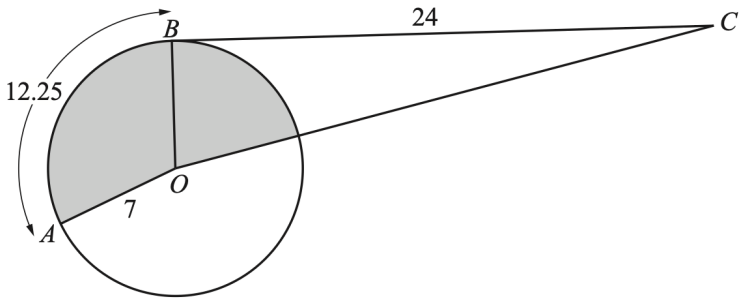
[2]

c. On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes.

[4]



4. In this question all lengths are in metres.



The diagram shows a circle, centre *O*, radius 7. The points *A* and *B* lie on the circumference of the circle. The line *BC* is a tangent to the circle at the point *B* such that the length of *BC* is 24. The length of the minor arc *AB* is 12.25.

a. Find the obtuse angle *AOB*, giving your answer in radians.

[2]

b. Find the perimeter of the shaded region.

[4]

c. Find the area of the shaded region.

[2]

5. The points P and Q have coordinates (5, -12) and (15, -6) respectively.

The point R lies on the line l, the perpendicular bisector of the line PQ.

The x-coordinate of R is 7. Find the y-coordinate of R.

[4]

6. Solve the following inequality.

$(2x + 3)(x - 4) > (3x + 4)(x - 1)$

[5]

7. Solve the equation $lg(2x - 1) + lg(x + 2) = 2 - lg4$.

[5]

8. The line $y = kx + 6$ intersects the curve $y = x^3 - 4x^2 + 3kx + 2$ at the point where $x = 2$.

(a) Find the value of k.

[2]

(b) Show that, for this value of k, the line cuts the curve only once.

[4]

9. Write $\frac{\sqrt{(9p^2q)}\times r^{-3}}{(2p)^3 q^{-1}\sqrt[5]{r}}$ in the form $kp^a q^b r^c$, where k, a, b and c are constants.

[4]

10. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Find the *x*-coordinates of the points of intersection of the curves

$y = 7x^3 - 7x^2 - 17x - 4$ and $y = x^3 - 2x^2 - 4x - 16$.

[5]